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Constraints on Holographic Cosmological Models from Gamma Ray Bursts

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Abstract

We use Gamma Ray Bursts (GRBs) data from *Y. Wang (2008)* to put additional constraints on a set of cosmological dark energy models based on the holographic principle. GRBs are among the most complex and energetic astrophysical events known in the universe offering us the opportunity to obtain information from the history of cosmic expansion up to about redshift of $z \sim 6$. These astrophysical objects provide us a complementary observational test to determine the nature of dark energy by complementing the information of data from Supernovas (e.g. Union 2.1 compilation). We found that the Λ CDM model gives the best fit to the observational data, although our statistical analysis (ΔAIC and ΔBIC) shows that the models studied in this work ("*Hubble Radius Scale*" and "*Ricci Scale Q*") have a reasonable agreement with respect to the most successful, except for the "*Ricci Scale CPL*" and "*Future Event Horizon*" models, which can be ruled out by the present study. However, these results reflect the importance of GRBs measurements to provide additional observational constraints to alternative cosmological models, which are mandatory to clarify the way in which the paradigm of dark energy or any alternative model is correct.

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1 Introduction

In order to explain the current acceleration of the universe, the fine-tuning problem of the value of Λ and the cosmic coincidence problem, different alternative models have been proposed. In framing the question of the nature of dark energy there are two directions. The first is to assume a new type of component of energy density, which may be a fluid of constant density or dynamic. The other direction is to modify the Einstein's equations thinking that the metric is inappropriate or that gravity works differently on large scales. The observational tests are of great importance to discriminate between these scenarios [1]. The holographic dark energy is one dynamical DE model proposed in the context of quantum gravity, so called *holographic principle*, which arose from black hole and string theories [2]. The holographic principle states that the number of degrees of freedom of a physical system, apart from being constrained by an infrared cutoff, should be finite and should scale with its bounding area rather than with its volume. Specifically, it is derived with the help of the entropy-area relation of thermodynamics of black hole horizons in general relativity, which is also known as the Bekenstein-Hawking entropy bound, i.e., $S \simeq M_p^2 L^2$, where S is the maximum entropy of the system of length L and $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass. This principle can be applied to the dynamics of the universe, where L may be associated with a cosmological scale and its energy density as:

$$\rho_H = \frac{3c^2 M_p^2}{L^2}. \quad (1)$$

If for example L is the Hubble's radius, which represents the current size of the universe, then ρ_H represents the holographic dark energy density associated.

Our analysis begins with the holographic dark energy models (§ 2), following with the Gamma-Ray Bursts model and data (§ 3) and then finish with the results and discussion (§ 4).

2 Holographic Dark Energy Models

In this section we present some of the most popular holographic dark energy models reported in the literature.

The Friedmann equations for a spatially flat universe can be written as:

$$3H = 8\pi G (\rho_m + \rho_H), \quad (2)$$

where ρ_m is the energy density of the matter component and ρ_H is the holographic dark energy density. These components are related by an interaction Q term as:

$$\frac{d\rho_m}{dt} + 3H\rho_m = Q \quad \frac{d\rho_H}{dt} + 3H(1 + w_H)\rho_H = -Q, \quad (3)$$

where $w_H = p_H/\rho_H$ is the equation of state of the holographic dark energy density. The change rate of the Hubble parameter can be written as:

$$\frac{dH}{dt} = -\frac{3}{2}H^2 (1 + w^{eff}), \quad (4)$$

where $w^{eff} = w/(1+r)$ is the effective equation of state of the cosmic fluid and $r = \rho_m/\rho_H$ is the ratio of energy densities, which is related to the saturation parameter c^2 as $c^2(1+r) = 1$, which establishes that energy in a box of size L should not exceed the energy of the black hole of the same size, under the condition $L^3\rho_H \leq M_p^2 L$. Different scales lead to different cosmological models [2].

2.1 Λ CDM

We begin our analysis with the standard cosmological model. In this paradigm, the DE is provided by the cosmological constant Λ , with an EoS, such that $w = -1$. In this model the Friedmman equation $E^2(z, \Theta)$ for a flat universe is given by

$$E^2(z, \Theta) = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda, \quad (5)$$

where Ω_m and Ω_Λ are the density parameters for matter and dark energy respectively and $\Omega_r = \Omega_\gamma(1 + 0.2271N_{eff})$ is the radiation density parameter, where $\Omega_\gamma = 2.469 \times 10^{-5}h^{-2}$ is photon density parameter and $N_{eff} = 3.046$ is the effective number of neutrinos. The free parameters are h , Ω_m , Ω_Λ and the best fit is shown in Table 1,

Λ Cold Dark Matter model	
$h = 0.7009 \pm 0.0035$	$\Omega_\Lambda = 0.716 \pm 0.028$
$\Omega_m = 0.266 \pm 0.0042$	

Table 1: Best fit parameters with all data set to Λ CDM model.

where $h = H_0/100km.s^{-1}Mpc^{-1}$ is dimensionless Hubble parameter.

2.2 Hubble Radius Scale

In this model $L = H^{-1}$ and the dark energy density $\rho_H = 3c^2M_p^2H^2$ and the Friedmann equation can be written as:

$$E^2(z) = [(1 - 2q_0) + 2(1 + q_0)(1 + z)^{3n/2}]^{1/n} \left(\frac{1}{3}\right)^{1/n}, \quad (6)$$

where q_0 is the present value of the deceleration parameter. This model is similar to Λ CDM when $n = 2$. The free parameters are h , q_0 and n , whose best fit values are shown in the Table 2.

<i>Hubble Radius Scale</i>	
$h = 0.7004 \pm 0.0038$	$n = 1.71 \pm 0.20$
$q_0 = 0.569 \pm 0.047$	

Table 2: Best fit parameters with all data set to *HRS* model.

2.3 Future Event Horizon $\xi = 1$

With $L = R_E$ the holographic DE density is $\rho_H = 3c^2 M_p^2 R_E^{-2}$, where R_E is the future event horizon. The Friedmann equation is given by:

$$E^2(z) = (1 + z)^{3/2-1/c} \sqrt{\frac{1 + r_0(1 + z)}{r_0 + 1}} \left[\frac{\sqrt{r_0(1 + z) + 1} - 1}{\sqrt{r_0 + 1} + 1} \right]^{2/c}, \quad (7)$$

where $R_E = c\sqrt{(1 + r)H^{-1}}$ and $r_0 = \Omega_0/(1 - \Omega_0)$. The best fit values of the free parameters h , r_0 and c are given in the Table 3.

<i>Future Event Horizon $\xi = 1$</i>	
$h = 0.6799 \pm 0.0025$	$r_0 = 0.322 \pm 0.032$
$c = 1.046 \pm 0.017$	

Table 3: Best fit parameters with all data set to *FEH* model.

2.4 Ricci Scale CPL

The Ricci scalar $R = 6(2H^2 + \dot{H})$ is relate to the cutoff-scale through $L^2 = 6/R$ and the energy:

$$\rho_H = 3c^2 M_p^2 \frac{R}{6} = \alpha (2H^2 + \dot{H}), \quad (8)$$

where $\alpha = 3c^2/8\pi G$. If we use the CPL parameterization $w(a) = w_0 + (1 - a)w_1$, the Friedmann equation can be written as:

$$E^2(z, \Theta) = (1+z)^{\frac{3}{2} \frac{1+r_0+w_0+4w_1}{1+r_0+3w_1}} \left[\frac{1+r_0+3w_1 \left(\frac{z}{1+z} \right)}{1+r_0} \right]^{-\frac{1}{2} \frac{1+r_0+3w_0}{1+r_0+3w_1}}. \quad (9)$$

The free parameters of this model are h , r_0 , w_0 and w_1 . The best fit is given in the Table 4.

<i>Ricci Scale CPL</i>	
$h = 0.6518 \pm 0.0021$	$r_0 = 9.39^{+0.51}_{-0.47}$
$w_0 = -2.64^{+0.49}_{-0.55}$	$w_1 = 0.46^{+0.34}_{-0.33}$

Table 4: Best fit parameters with all data set to *RSCPL* model.

2.5 Ricci Scale Q

If the interaction term is given by $Q = 3H\beta\rho_H$, then

$$\beta = \frac{1}{1+r} \left[rw - \frac{\dot{w}}{H} \right] \quad (10)$$

and the EoS is given by:

$$w = -\frac{1}{6} \frac{u - s - (u+s)Aa^s}{1 - Aa^s} \quad (11)$$

where $u \equiv r_0 - 3w_0 + 3\beta$, $v \equiv r_0 + 3w_0 + 3\beta$, $s \equiv (u^2 - 12\beta(1+r_0-3w_0))^{1/2}$ y $A \equiv (v-s)/(v+s)$. The Friedmann equation can be written as:

$$E^2(z, \Theta) = \left[\frac{n(1+z)^{-s} - m}{n - m} \right]^{\frac{3}{2} \frac{lm-kn}{mns}} (1+z)^{\frac{3}{2} \left(1 - \frac{k}{m} \right)}, \quad (12)$$

such that $m \equiv 1 + r_0 - 1/2(v-s)$, $n \equiv [1 + r_0 - 1/2(v+s)] A$, $k \equiv 1/6(u-s)$ y $l \equiv 1/6(u+s)A$. The free parameters are h , r_0 , w_0 and β , whose best fit is shown in the Table 5.

<i>Ricci Scale Q</i>	
$h = 0.6999 \pm 0.0038$	$r_0 = 0.201^{+0.025}_{-0.022}$
$w_0 = -0.842^{+0.055}_{-0.056}$	$\beta = -0.011^{+0.015}_{-0.019}$

Table 5: Best fit parameters with all data set to *RSCPL* model.

3 Gamma-Ray Bursts

In this section we present a brief introduction of GRBs as astrophysical objects and its later use in cosmology.

3.1 GRBs Model

Gamma-ray bursts (GRBs) are the most luminous astrophysical events observable today, because they are at cosmological distances. The duration of a gamma-ray burst is typically a few seconds, but can range from a few milliseconds to several minutes. The initial burst at gamma-ray wavelengths is usually followed by a longer lived afterglow at longer wavelengths (X-ray, ultraviolet, optical, infrared, and radio). Gamma-ray bursts have been detected by orbiting satellites about two to three times per week. Most observed GRBs appear to be collimated emissions caused by the collapse of the core of a rapidly rotating, high-mass star into a black hole. At least once a day, a powerful source of gamma rays temporarily appears into the sky in an unpredictable location and later disappears, which last for milliseconds to minutes. In the location of the gamma ray event is usually observed a dominant afterglow in X-rays, optical and radio after long decays.

3.2 GRBs Data

We use GRB data in the form of the model-independent distance from Wang (2008) [6], which were derived from the data of 69 GRBs with $0.17 \leq z \leq 6.6$ from Schaefer (2007). The GRB data are included in our analysis by adding the following term to the given model:

$$\chi_{GRB}^2 = [\Delta \bar{r}_p(z_i)] \cdot (C_{GRB}^{-1})_{ij} \cdot [\Delta \bar{r}_p(z_i)], \quad (13)$$

where $\Delta \bar{r}_p(z_i) = \bar{r}_p^{data}(z_i) - \bar{r}_p(z_i)$ and $\bar{r}_p(z_i)$ is given by

$$\bar{r}_p(z_i) = \frac{r_p(z)}{r_p(0.17)} \quad (14)$$

where

$$r_p(z) = \frac{(1+z)^{1/2} H_0}{z} \frac{1}{ch} r(z) \quad (15)$$

and $r(z)$ is the comoving distance at z . The covariance matrix is given by:

$$C_{ij}^{grb} = \sigma(\bar{r}_p(z_i)) \sigma(\bar{r}_p(z_j)) \bar{C}_{ij}^{grb} \quad (16)$$

where \bar{C}_{ij}^{grb} is the normalized covariance matrix:

Data point	(z)	$\bar{r}_p(z_i)^{dat}$	$\sigma(\bar{r}_p(z_i))^+$	$\sigma(\bar{r}_p(z_i))^-$
0	0.17	1.0000	-	-
1	1.036	0.9416	0.1688	0.1710
2	1.902	1.0011	0.1395	0.1409
3	2.768	0.9604	0.1801	0.1785
4	3.634	1.0598	0.1907	0.1882
5	4.500	1.0163	0.2555	0.2559
6	6.600	1.0862	0.3339	0.3434

Table 6: Distances independent model GRBs.

$$\bar{C}_{ij}^{grb} = \begin{pmatrix} 1.0000 & & & & & \\ 0.7056 & 1.0000 & & & & \\ 0.7965 & 0.5653 & 1.0000 & & & \\ 0.6928 & 0.6449 & 0.5521 & 1.0000 & & \\ 0.5941 & 0.4601 & 0.5526 & 0.4271 & 1.0000 & \\ 0.5169 & 0.4376 & 0.4153 & 0.4242 & 0.2999 & 1.0000 \end{pmatrix} \quad (17)$$

and

$$\sigma(\bar{r}_p) = \begin{cases} \sigma(\bar{r}_p(z_i))^+, & \text{if } \bar{r}_p(z_i) \geq \bar{r}_p(z_i)^{dat} \\ \sigma(\bar{r}_p(z_i))-, & \text{if } \bar{r}_p(z_i) < \bar{r}_p(z_i)^{dat} \end{cases} \quad (18)$$

where $\sigma(\bar{r}_p(z_i))^+$ and $\sigma(\bar{r}_p(z_i))^-$, are 68% C.L errors given in the Table 6. As complementary tests we use SNIa (580-Data point), CMB (1-Data point) and BAO (1-Data point) [4] (See Appendix).

4 Results and Discussion

In this section we perform the statistical analysis, where we implemented the maximum likelihood criterion to get the best settings for each model and used the AIC and BIC criterion to discriminate between the different models.

The maximum likelihood estimate for the best fit parameters is:

$$\mathcal{L}_{max} = \exp \left[-\frac{1}{2} \chi_{min}^2 \right] \quad (19)$$

If \mathcal{L}_{max} has a Gaussian errors distribution, then $\chi_{min}^2 = -2 \ln \mathcal{L}_{max}$, So, for our analysis:

$$\chi_{min}^2 = \chi_{GRBs}^2 + \chi_{SNIa}^2 + \chi_{CMB}^2 + \chi_{BAO}^2. \quad (20)$$

Model	χ^2_{min}	AIC	BIC	ΔAIC	ΔBIC
ΛCDM	608.5	614.5	627.6	0.0	0.0
<i>Hubble Radius S.</i>	609.7	615.7	628.8	1.2	1.2
<i>Future Event H.</i>	657.3	663.3	676.4	48.8	48.8
<i>Ricci scale CPL</i>	917.3	925.3	924.8	310.8	315.2
<i>Ricci scale Q</i>	609.8	617.8	635.3	3.3	7.7

Table 7: AIC and BIC analysis to diferent dark energy models using all data sets.

In the Figure 1 we show the diagrams of statistical confidence at 1σ , 2σ and 3σ for different cosmological models and several parameter space, from a joint analysis of 69 GRBs (independent-model 6-Data point), SNIa (580-Data point), CMB (1-Data point), BAO (1-Data point) [4] [2].

In this paper we use the Akaike and Bayesian information criterion (AIC, BIC), which allow to compare cosmological models with different degrees of freedom, with respect to the observational evidence and the set of parameters [5]. The AIC and BIC can be calculated as:

$$AIC = -2 \ln \mathcal{L}_{max} + 2k, \quad (21)$$

$$BIC = -2 \ln \mathcal{L}_{max} + k \ln N, \quad (22)$$

where \mathcal{L}_{max} is the maximum likelihood of the model under consideration, k is the number of parameters. BIC imposes a strict penalty against extra parameters for any set with N data. The prepered model is that which minimizes AIC and BIC, however, only the relative values between the different models is important [3]. The results are shown in the Table 7.

5 Conclusion

We implement GRBs data model-independent from *Y. Wang (2008)* to complement SNIa Union2.1 sample to high redshift. We found that model-independent GRBs provide additional observational constraints to holographic dark energy models. In addition to the data of GRBs we use data from SNIa, BAO and CMB as usual cosmological tests.

Our analysis shows that the ΛCDM model is preferred by the ΔAIC and ΔBIC criterion. The "*Hubble Radius Scale*" and "*Ricci Scale Q*" models show an interesting agreement with the observational data. The "*Ricci Scale CPL*" and "*Future Event Horizon*" models can be ruled out by the present analysis.

Finally we want to highlight the importance of deepening in the development of unified models of GRBs, given the obvious importance of these objects in the era of cosmology accuracy, which help to shed light on the dark energy paradigm.

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⁴<http://gravityschool.uniandes.edu.co/index.php/en/speakers>

⁵<http://ifa.uv.cl/index.php/es/2-uncategorised/83-dr-gael-foeex>

Appendix

SNIa

Here, we use the Union 2.1 sample which contains 580 data points. The SNIa data give the luminosity distance $d_L(z) = (1+z)r(z)$. We fit the SNIa with the cosmological model by minimizing the χ^2 value defined by

$$\chi_{SNIa}^2 = \sum_{i=1}^{580} \frac{[\mu(z_i) - \mu_{obs}(z_i)]^2}{\sigma_{\mu_i}^2}, \quad (23)$$

where $\mu(z) \equiv 5 \log_{10}[d_L(z)/\text{Mpc}] + 25$ is the theoretical value of the distance modulus, and μ_{obs} the observed one.

CMB

We also include CMB information by using the WMAP data. The χ_{cmb}^2 for the CMB data is constructed as:

$$\chi_{cmb}^2 = \frac{(1.7246 - R)^2}{0.03^2}. \quad (24)$$

Here R is “shift parameter”, defined as:

$$R = \frac{\sqrt{\Omega_m}}{c(1+z_*)} D_L(z). \quad (25)$$

where $d_L(z) = D_L(z)/H_0$ and the redshift of decoupling z_* is $z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}]$ and

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}. \quad (26)$$

BAO

Similarly, for the DR7 BAO data, the χ^2 can be expressed as:

$$\chi_{6dFGS}^2 = \left(\frac{d_z - 0.469}{0.017} \right)^2. \quad (27)$$

where $d_z = r_s(z_d)/D_V(z)$ denotes the distance ratio. Here, $r_s(z_d)$ is the comoving sound horizon at the baryon drag epoch ($z_d = 0.35$) and $D_V(z)$ is the acoustic scale.

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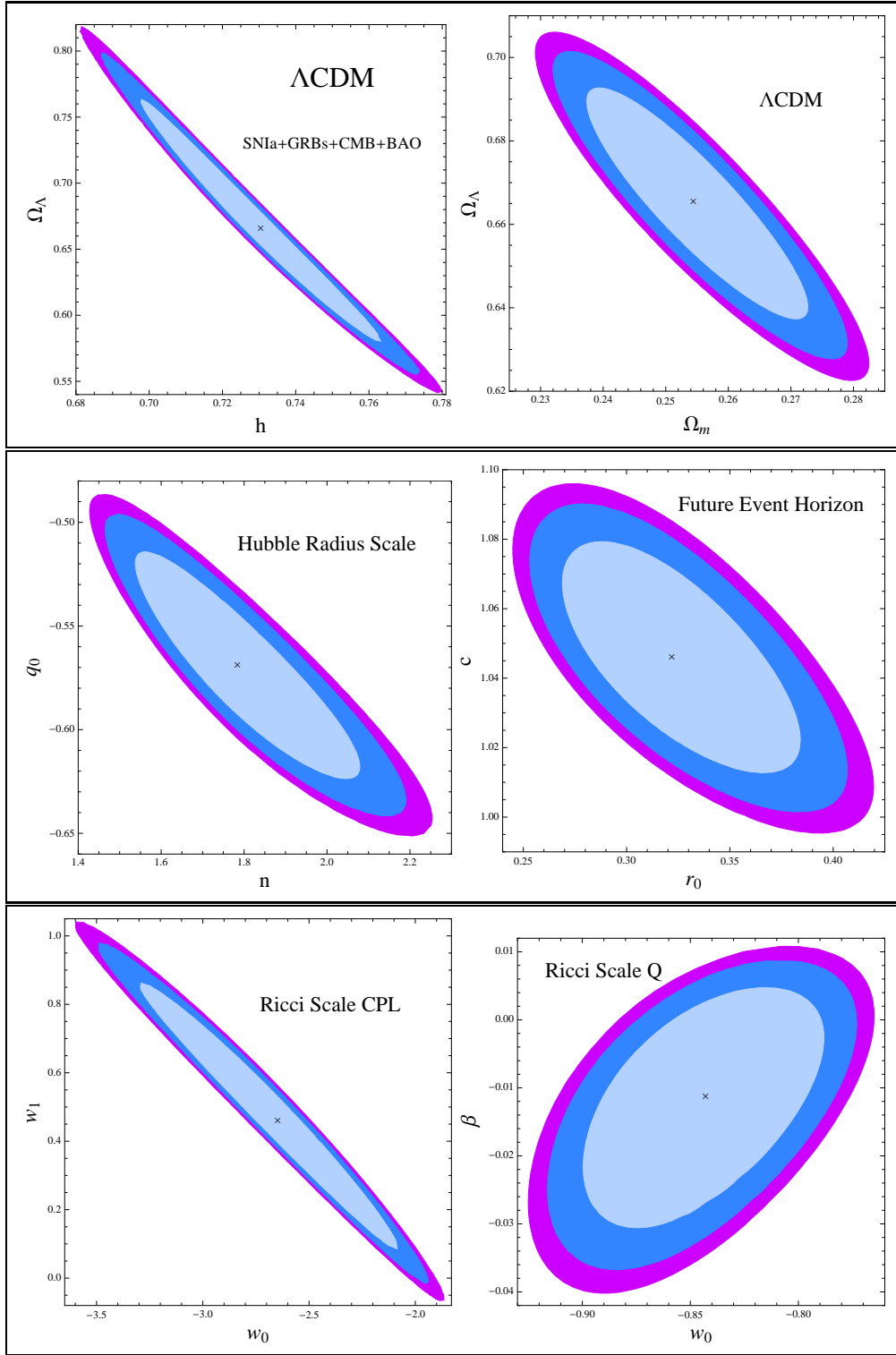


Figure 1: Diagrams of statistical confidence marginalizing different cosmological parameters at 1 σ , 2 σ and 3 σ for different cosmological models.